On the Analysis of Single- and Multiple-Step Discontinuities for a Shielded Three-Layer Coplanar Waveguide

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Abstract— **Single- and multiple-step discontinuities for a shielded three-layer coplanar waveguide (CPW) are studied. The mode matching procedure is employed to obtain the scattering (S) parameters of the discontinuities. The analysis is validated through a comparison of the calculated S-parameters of a single step discontinuity for a shielded single-layer CPW and those published previously. Calculated S-parameters for various single, double, and triple step discontinuities are presented. Effect of the modal orthogonality criterion on the discontinuity S-parameters is given. Extensive investigation of the numerical convergence of the S-parameters is also described.**

I. Introduction

TRADITIONALLY, MICROSTRIPLINE has been widely used as the primary transmission line for hybrid and monolithic microwave and millimeter integrated circuits (MICs and MMICs). Recently, however, there has been considerable interest and use of coplanar waveguide for MICs and MMICs due to its attractive features. The use of CPW can eliminate via holes in connecting circuit elements to ground, allow easy realizations of compact balanced circuits, and reduce cross talk between lines.

Without any question, the analysis of CPW discontinuities plays the most important role in the design and analysis of CPW MICs and MMICs. In order to achieve high-performance, low-cost microwave components employing CPW, highly accurate analysis of CPW discontinuities are needed. In comparison with microstrip discontinuities, activities on CPW discontinuity analysis have been sporadic, in spite of several attractive advantages of CPW. Very little work on CPW discontinuities has been done so far [1]–[8]. One of the most commonly encountered discontinuities in MICs and MMICs is the step discontinuity. A lumped element equivalent circuit model of a symmetric step in the center conductor of a conventional nonconductor backed CPW, based on measured scattering (S) parameter, has been developed [4], [5]. Full-wave modal analyses of a single-step discontinuity [6], [7] and cascaded junction discontinuities [8] for a shielded single layer CPW has also recently been reported. However, analysis for single- and multiple-step discontinuities in a shielded three-layer CPW structure has not yet been attempted. As compared to the single-layer CPW, the three-layer structure can suppress the leaky wave as well as control the fundamental-mode frequency range by using appropriate dielectric substrates for the top and the bottom layers.

In this paper, we present the analysis of single and multiple step discontinuities for a shielded three-layer CPW using the full-wave mode matching technique [9], utilizing the eigenmodes obtained based on the spectral domain approach [10]. Various numerical results for the S-parameters of single and multiple step discontinuities in CPW are presented. Effect of the modal orthogonality criterion on the numerical values of the discontinuity S-parameters is investigated. Extensive investigation of the convergence for the discontinuities' S-parameters is also given. S-parameter data of a single-step discontinuity for a shielded single-layer CPW, generated using the developed analysis, agree well with the published data [6], [7].

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II. Modal Analysis of Step Discontinuities

Fig. 1 shows cross-section of the three-layer CPW, enclosed in a perfectly conducting shield, and its step discontinuities, classified as single step [Fig. 1(b)], double step [Fig. 1(c)], and multiple step [Fig. 1(d)]. Both the ground planes and center strip are assumed to be perfect conductors and infinitely thin, and the dielectric substrates are assumed to be loss less. The discontinuities are very general in that the widths of the center conducting strip and ground planes can be changed separately or simultaneously. Both symmetrical and asymmetrical changes can also be accommodated.

A. Single-Step Discontinuity

Fig. 1(b) shows the considered single step discontinuity, where region b is assumed to be extended to infinity. In using the mode-matching technique, the hybrid modes, including the propagation and evanescent modes, on the left-hand side (region a) and right-hand side (region b) of the plane of the discontinuity ($z = 0$) are first determined using the spectral domain technique [10]. Assume a wave with a unit modal voltage is incident from region a. The transverse electric and magnetic fields on regions a and b, at the plane of the discontinuity, are then expressed in terms of these eigenmodes and matched across the discontinuity, resulting in the following

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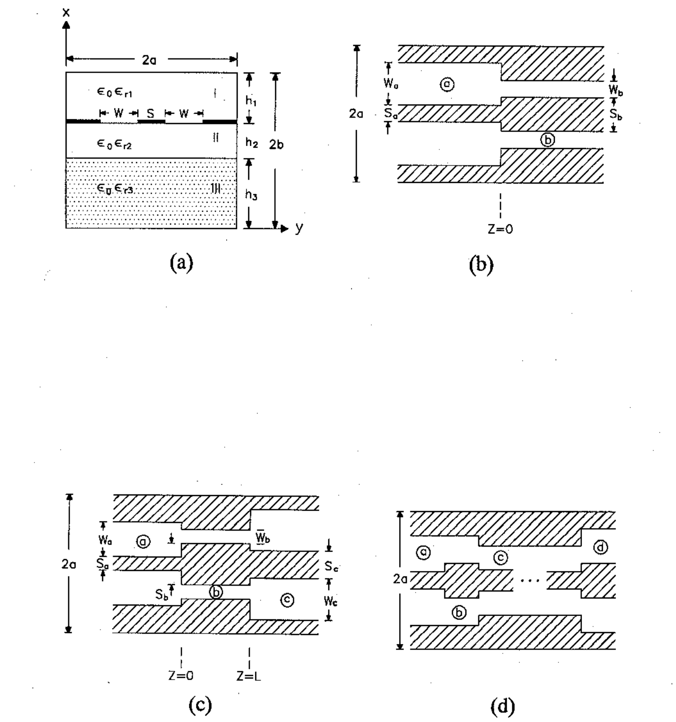

Fig. 1. A shielded three-layer CPW cross-section (a) and single- (b), double- (c) and multiple- (d) step discontinuities.

mode-matching equation:

$$(1+\rho)\bar{e}_1^a + \sum_{i=2}^{\infty} a_i \bar{e}_i^a = \sum_{j=1}^{\infty} b_j \bar{e}_j^b \tag{1a}$$

$$(1-\rho)\bar{h}_1^a - \sum_{i=2}^{\infty} a_i \bar{h}_i^a = \sum_{j=1}^{\infty} b_j \bar{h}_j^b \tag{1b}$$

where $\bar{e}^a$, $\bar{h}^a$ and $\bar{e}^b$, $\bar{h}^b$ are the normalized electric and magnetic field vectors in region a and b, respectively, with $\bar{e}_1^a$ and $\bar{h}_1^a$ representing the respective electric and magnetic fields associated with the reflection coefficient ρ of the incident mode; a_i and b_j are the complex amplitude coefficients of modes i and j scattering into regions a and b from the incident mode at the discontinuity, respectively.

In order to solve (1), we introduce an inner product defined by

$$I_{nm}^{\gamma\lambda} = \int_S \bar{e}_n^{\gamma} \times \bar{h}_m^{\lambda} \cdot \overline{dS} \tag{2}$$

$\gamma = a$ or b and $\lambda = a$ or b. S in the guide cross-section. By taking the inner products of (1a) and (1b) with $\bar{h}_m^b$ and $\bar{e}_n^a$, respectively, and truncating the infinite series to N_a and N_b, which signify the number of eigenmodes in regions a and b, respectively, one obtains the following system of $(N_a + N_b)$ linear equations:

$$(1+\rho)I_{1m}^{ab} + \sum_{i=2}^{N_a} a_i I_{im}^{ab} - \sum_{j=1}^{N_b} b_j I_{jm}^{bb} = 0$$
$$m = 1, \cdots, N_b \tag{3a}$$

$$(1-\rho)I_{n1}^{aa} - \sum_{i=2}^{N_a} a_i I_{ni}^{aa} - \sum_{j=1}^{N_b} b_j I_{nj}^{ab} = 0$$
$$n = 1, \cdots, N_a. \tag{3b}$$

Note that, while taking the inner products, the boundary enlargement and reduction considerations [9] have been implemented for improving the convergence. The aforementioned equations can be immediately solved to obtain the solutions for the $(N_a + N_b)$ unknowns ρ, a_i $(i = 2, 3, \cdots, N_a)$, and b_j $(j = 1, 2, \cdots, N_b)$, which characterize the S-parameters of the discontinuity.

B. Double-Step Discontinuity

For the double-step discontinuity [Fig. 1(c)], the transverse electric and magnetic fields in regions a $(z \le 0)$, b $(0 \le z \le L)$, and c $(z \ge L)$, at the planes of the discontinuities $(z = 0, L)$ are expressed as a superposition of the incident, reflected, and transmitted eigenmodes. Assume a wave of unit modal voltage is incident from region a. By imposing the continuity of the transverse fields across the discontinuities, the mode-matching equations at the $z = 0$ plane and $z = L$ plane can be written as

$$(1+\rho)\bar{e}_1^a + \sum_{p=2}^{\infty} a_p \bar{e}_p^a = \sum_{q=1}^{\infty} b_q \bar{e}_q^b + \sum_{r=1}^{\infty} c_r \bar{e}_r^b \cdot \exp\left(-j\beta_r^b L\right) \tag{4a}$$

$$(1-\rho)\bar{h}_1^a - \sum_{p=2}^{\infty} a_p \bar{h}_p^a = \sum_{q=1}^{\infty} b_q \bar{h}_q^b - \sum_{r=1}^{\infty} c_r \bar{h}_r^b \cdot \exp\left(-j\beta_r^b L\right) \tag{4b}$$

and

$$\sum_{q=1}^{\infty} b_q \bar{e}_q^b \exp\left(-j\beta_q^b L\right) + \sum_{r=1}^{\infty} c_r \bar{e}_r^b = \sum_{s=1}^{\infty} d_s \bar{e}_s^c \tag{4c}$$

$$\sum_{q=1}^{\infty} b_q \bar{h}_q^b \exp\left(-j\beta_q^b L\right) - \sum_{r=1}^{\infty} c_r \bar{h}_r^b = \sum_{s=1}^{\infty} d_s \bar{h}_s^c \tag{4d}$$

respectively, where $\bar{e}^n$ and $\bar{h}^n$ $(n = a, b, c)$ represent the normalized electric and magnetic field vectors in regions n, respectively; $\bar{e}_1^a$ and $\bar{h}_1^a$ are the normalized transverse electric and magnetic fields of the incident mode having reflection coefficients ρ; a_p and b_q are the complex coefficients of modes p and q scattering into regions a and b from the incident mode at the first discontinuity, respectively; c_r and d_s denote the complex coefficients of modes r and s scattering into regions b and c from the second discontinuity, respectively; β_r^b is the propagation constant of mode r in region b. Taking the inner products of (4) with $\bar{h}_k^a$, $\bar{e}_l^b$, and $\bar{h}_m^c$, respectively, in accordance with the inner product defined in (2) with γ and λ now denoting region a, b, or c, yields the following system of $(N_a + 2N_b + N_c)$ linear equations:

$$(1+\rho)I_{1k}^{aa} + \sum_{p=2}^{N_a} a_p I_{pk}^{aa} - \sum_{q=1}^{N_b} b_q I_{qk}^{ba} - \sum_{r=1}^{N_b} c_r I_{rk}^{ba} \exp\left(-j\beta_r^b L\right) = 0,$$
$$k = 1, \cdots, N_a \tag{5a}$$

$$(1-\rho)I_{l1}^{ba} - \sum_{p=2}^{N_a} a_p I_{lp}^{ba} - \sum_{q=1}^{N_b} b_q I_{lq}^{bb} + \sum_{r=1}^{N_b} c_r I_{lr}^{bb} \exp(-j\beta_r^b L) = 0, \quad l = 1, \cdots, N_b \tag{5b}$$

$$\sum_{q=1}^{N_b} b_q I_{qm}^{bc} \exp(-j\beta_q^b L) + \sum_{r=1}^{N_b} c_r I_{rm}^{bc} - \sum_{s=1}^{N_c} d_s I_{sm}^{cc} = 0, \quad m = 1, \cdots, N_c \tag{5c}$$

$$\sum_{q=1}^{N_b} b_q I_{nq}^{bb} \exp(-j\beta_q^b L) - \sum_{r=1}^{N_b} c_r I_{nr}^{bb} - \sum_{s=1}^{N_c} d_s I_{ns}^{bc} = 0, \quad n = 1, \cdots, N_b. \tag{5d}$$

The infinite series have been truncated to N_a, N_b, and N_c that approximate the number of eigenmode in regions a, b, and c, respectively. These equations can now be solved to determine the solutions for the unknowns, ρ, a_p $(p = 2, 3, \cdots, N_a)$, b_q, and c_r $(q.r = 1, 2, \cdots, N_b)$, and d_s $(s = 1, 2, \cdots, N_c)$.

C. Multiple-Step Discontinuity

Analysis for the multiple step problem of Fig. 1(d) is very similar. By extending the foregoing theory, the composite generalized scattering matrix of the multiple step can be obtained, taking into account the interactions between the junctions via the fundamental as well as higher order modes.

It is well known that eigenmodes of a waveguide with perfectly conducting walls should satisfy the following modal orthogonality criterion.

$$I_{ij}^{nn} = \int_S \overline{e}_i^n \times \overline{h}_j^n \cdot \overline{dS} = \delta_{ij} \tag{6}$$

where δ_{ij} is Kronecker delta function and $n = a, b, c,$ or d. So this should serve as a test for the accuracy of the numerically computed modes.

D. Numerical Results

In order to verify the developed analysis of step discontinuities, numerical computations have been performed for a single-step discontinuity in a shielded single-layer CPW and compared with published data. Various numerical results for S-parameters of single, double, and triple steps of shielded three-layer CPWs were also generated to demonstrate our analysis.

Results for a CPW single step discontinuity presented in [6], [7] were compared with our results. Almost complete agreement in magnitudes and phases of the S-parameters was observed.

In Fig. 2, we show variations of calculated S-parameters versus the frequency for a double three-layer CPW step. In Fig. 3, we plot the magnitudes of a double CPW step as a function of the mid-region length L at 20 GHz. It is apparent, because of the identical regions employed in a and c, that a reduction in L increases the transmission.

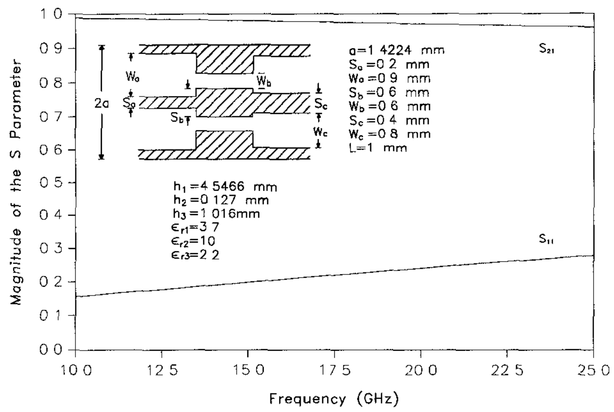

Fig. 2. Calculated magnitudes of S_{11} and S_{21} of shielded three-layer CPW double step.

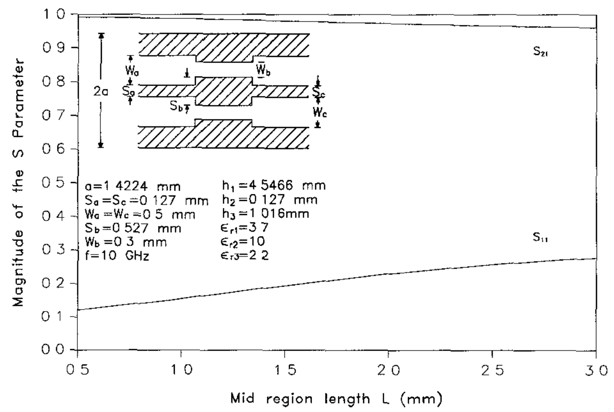

Fig. 3. Variation of the magnitudes of S_{11} and S_{21} as functions of the mid-region length L for a CPW double step.

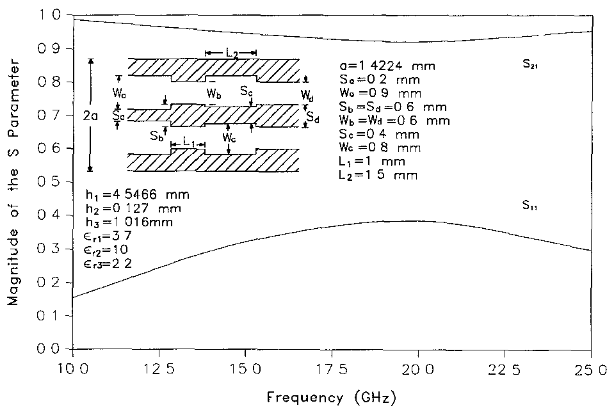

Fig. 4. Calculated magnitudes of S_{11} and S_{21} of a shielded three-layer CPW triple step.

Finally, the computed magnitudes of the S-parameters for a triple step in a shielded three-layer CPW are plotted in Fig. 4.

In order to assess the accuracy of the obtained results, we have investigated and verified numerically the validity of the power conservation $(|S_{11}|^2 + |S_{21}|^2 = 1)$, the boundary conditions on the $y = h_2 + h_3$ plane, the orthogonality condition, and the convergence of the S-parameters.

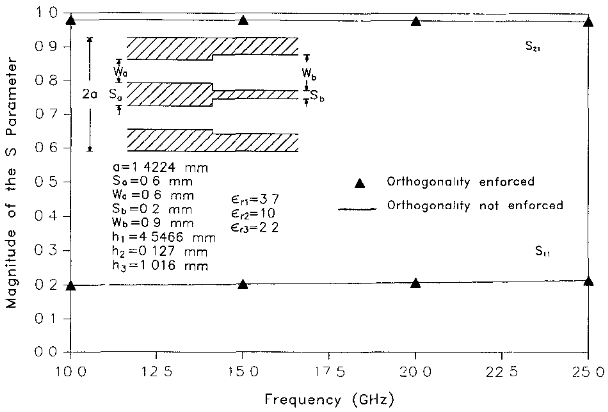

Fig. 5. Comparison of the calculated magnitudes of S_{11} and S_{21} for a shielded three-layer CPW single step with and without the orthogonality enforced.

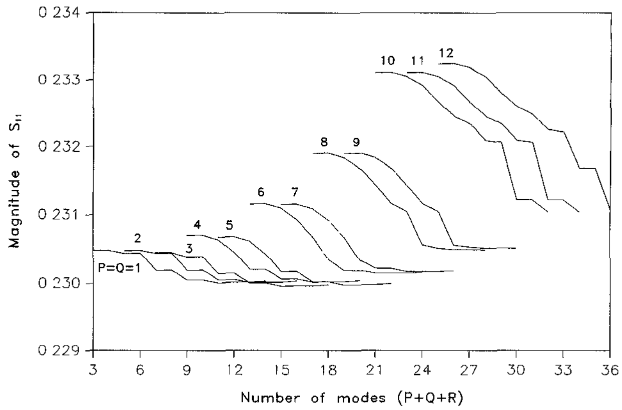

Fig. 6. Relative convergence of the magnitude S_{11} for the CPW of Fig. 2 with $L = 2$ mm, as a function of the total number of modes used. $P =$ number of modes in region a, $Q =$ number of modes in region c, and $R =$ number of modes in region b. $f = 10$ GHz.

As the CPW eigenmode calculations are approximate due to the necessary numerical truncations, deviation from the orthogonality criterion sometimes results, as expected. To illustrate this phenomena, we compare the S-parameters for a single CPW step, calculated with and without the assumption of the orthogonality condition, in Fig. 5. As the mode calculations are very accurate in this case, these two results have merged completely to one another. For poorly calculated eigenmodes slight variation between the two results is expected.

The numerical convergence for the S-parameters of the CPW step discontinuities as a function of the number of eigenmodes considered is now investigated. An extensive study on the convergence of the magnitude of S_{11}, performed on several single-step discontinuous structures, has shown that a good three-digit agreement is achieved when using more than 5 eigenmodes in both of the regions. The convergence phenomenon of the S-parameters for a CPW double-step structure has also been studied. Fig. 6 shows the convergence results for the magnitude of S_{11}. For each curve, the number of modes P and Q in the two outer regions are kept equal and constant, while that in the middle R is varied. It is readily seen that the numerical convergence is approached as the number of the mid-region modes is increased. The number of modes in the two outer regions has little effect on the convergence of the S-parameters. This result is not unexpected due to the fact that interactions between modes in the outer regions occur in the mid-region, resulting in a rapid variation of current in the double-discontinuous structure. A remark needs to be made at this point is that the convergence criterion may vary largely form one structure to another for a double step as well as for multiple steps. In particular, for a small mid-region length, there will be more interactions between modes in the mid-region since evanescent modes created in one junction will be able to reach the other junctions before decaying completely. Consequently, more modes need to be employed in the mid-region for the convergence to take place. In our double-step calculations, we used 6 modes in the outer regions ($N_a = N_c = 6$) and 12 modes in the mid-region ($N_b = 12$). For the triple-step calculations, we also employed 6 modes in the two outer regions and 12 modes in the two mid-regions.

III. Conclusions

A modal analysis for single- and multiple-step discontinuities of a shielded three-layer CPW has been reported. The analyzed step discontinuities are arbitrary in which either the center strip width, the ground planes, or both can be varied and both symmetrical and asymmetrical changes in widths are considered. Very good agreement between our calculated results and those published for a single shielded single-layer CPW step has been observed. The modal orthogonality criterion has been studied and found to affect the S-parameter numerical results, power conservation, and convergence. The developed step discontinuity analysis for the shielded three-layer CPW structure should be very useful for the design of CPW MICs and MMICs.

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